

# **BRaille PIATHEMATIC NOTATION**

**Document produced by  
the Commission for the Evolution of French Braille**

**under the auspices of the  
Ministry of Health and Solidarity Directorate General  
for Social Action**

**First edition January 2007**

Printed by  
the Department of Transcription and Adapted Publishing  
National Institute for Young Blind People  
56, boulevard des Invalides  
75007 Paris

**Phone: 01 44 49 35 74  
Fax: 01 40 61 13 40 E-mail:  
[dtea@inja.fr](mailto:dtea@inja.fr)  
Website: <http://www.inja.fr>**

**and**

**the Valentin Haüy Association  
5 rue Duroc  
75007 Paris**

**Phone: 01 44 49 27 27  
Fax: +33 (0)1 44 49 27 10  
Email: [imprimerie@avh.asso.fr](mailto:imprimerie@avh.asso.fr)  
Website: <http://www.avh.asso.fr>**

---

# Summary

|                                                                   |     |
|-------------------------------------------------------------------|-----|
| MATHEMATICAL NOTATION IN BRAILLE.....                             | 4   |
| General .....                                                     | 4   |
| Conventions used in this document .. .. .                         | 5   |
| INTRODUCTION .....                                                | 6   |
| I. Braille writing ... .. t. ....                                 | 6   |
| II. Transcription of mathematics .. . . .                         | 7   |
| 1° General .....                                                  | 7   |
| 2° Formation of Braille symbols. ....                             | 8   |
| 3° Processing of mathematical expressions .....                   | 8   |
| 4° Capital letters in mathematics. ....                           | 8   |
| 5° Breaking up a mathematical expression. ....                    | 9   |
| 6° Spaces in a formula. ... .. .                                  | 9   |
| 7° Punctuation. ....                                              | 9   |
| 8° Use of abbreviations. ....                                     | 9   |
| t° Manual correction. ....                                        | i i |
| 1. Numbers and infinity sign. .... .                              | 12  |
| 2. Different types of letters. . . . .                            | 13  |
| 2.1 Round letters. ....                                           | 3   |
| 2.2 "Illuminated" letters .....                                   | 13  |
| 2.3 Greek letters. ....                                           | 13  |
| 2.4 Hebrew letters. ....                                          | is  |
| 3. Signs of operations. ... ..                                    | 7   |
| 4. Signs of equality and inequality . . . . .                     | 19  |
| 5. Parentheses, braces, and square brackets .....                 | 21  |
| 6. Ensemble notations. ....                                       | 23  |
| 6.1 Sets of numbers. ....                                         | 23  |
| 6.2 Various notations. ....                                       | 23  |
| 6.3 Intervals. ....                                               | 24  |
| 6.4 Examples. ....                                                | 24  |
| 7. Divisions, fractions. ....                                     | 25  |
| 8. Blocks. ....                                                   | 26  |
| 9. Iriclices. ....                                                | 28  |
| 9.1 Mathematical emphases. . . . .                                | 28  |
| 9.2 Lower index. .... .                                           | 28  |
| 9.3 Upper index, exponent. ....                                   | 29  |
| 9.4 Subscripts and superscripts .....                             | 30  |
| 9.5 Indices placed to the left of a quantity or prefixes .. . . . | 30  |
| 9.6 Superscript or subscript indices .. . . .                     | 32  |
| 10. Radicals .....                                                | 33  |
| 11. Arrows. .... .                                                | 34  |
| 12. Signed signs, unsigned signs, geometry .....                  | 36  |
| 12.1 Signs above .. . . .                                         | 36  |
| 12.2 Signatures. ....                                             | 38  |
| 12.3 Geometric signs .....                                        | 39  |
| 13. Frames, vertical bars .....                                   | 40  |

|                                            |    |
|--------------------------------------------|----|
| 13.1 Notation .....                        | 40 |
| 13.2 Absolute value, modulus, norm. .... . | 40 |

|                                                                                                |    |
|------------------------------------------------------------------------------------------------|----|
| 13.3 Determinants and matrices. ....                                                           | 41 |
| 13.4 Systems .. ...                                                                            | 42 |
| 14. Functions.....                                                                             | 43 |
| 14.1 Notation .....                                                                            | 43 |
| 14.2 Limits. ....                                                                              | 44 |
| 14.3 Sign tables, variation tables .....                                                       | 45 |
| 14.4 Integrals.....                                                                            | 48 |
| 14.5 Logarithms. ....                                                                          | 49 |
| 14.6 Trigonometry. ....                                                                        | 50 |
| 14.7 Hyperbolic lines. . . . .                                                                 | 52 |
| 15. Quantifiers and implication signs. ....                                                    | 53 |
| 16. Main units .....                                                                           | 55 |
| 16.1 General information. ....                                                                 | 35 |
| 16.2 Rating .....                                                                              | 56 |
| a) Decimal multiples and submultiples .....                                                    | S6 |
| b) Length.....t.....                                                                           | 57 |
| c) Area or surface area .....                                                                  | S7 |
| d) Agrarian measurements. ....                                                                 | 58 |
| e) Volume. ....                                                                                | 58 |
| <b>f) Capacité</b> .....                                                                       | 59 |
| g) Mass t.....                                                                                 | S9 |
| h) Coins.....                                                                                  | 60 |
| i) Calorimetry.....                                                                            | 60 |
| j) Work, energy .....                                                                          | 60 |
| k) Power ... ..                                                                                | i  |
| l) Force .t. ....                                                                              | i  |
| M) Pfr2SSIOFI ..... t.....                                                                     | 61 |
| n) Electricity, magnetism . ...                                                                | 62 |
| o) Optics .....                                                                                | 62 |
| p) Radioactivity. ....                                                                         | 63 |
| q) Angle and area .....                                                                        | 6s |
| r) Time, frequency.....                                                                        | 64 |
| s) Acoustics.....                                                                              | 64 |
| t) Temperature.....                                                                            | 64 |
| 16.3 Examples.. ..t. . . . .                                                                   | 63 |
| a) In full Braille .....                                                                       | 65 |
| b) In abbreviated Braille . . . . .                                                            | 66 |
| 16.4 Miscellaneous signs . . . . .                                                             | 67 |
| SUMMARY TABLE.....                                                                             | 69 |
| I. Value of keys before a letter .....                                                         | 69 |
| II. Symbols obtained by combining keys and numbers. ....                                       | 69 |
| III. Symbols obtained by combining keys and characters other than letters and numbers. . . . . | 72 |
| IV. Symbols obtained by combinations of some keys and letters. ....                            | 80 |
| V. Available symbols....                                                                       | i  |

# **BRAILLE MATHEMATICAL NOTATION**

## **General presentation**

**This mathematical notation, approved by the Commission for the Evolution of Braille Français at its plenary meeting on October 16, 2006, is applicable as of September 1<sup>st</sup> 2007.**

**It takes into account changes to the Braille Transcription Code for Printed Texts (1<sup>st</sup> same January 2006 edition), applicable in schools as of that date.**

**It replaces the 1971 (Braille version) and 1973 (printed version) editions, revised in 2001.**

**Jean-Marie CIERCO**

**President of the Commission for the Evolution of French Braille**

**Educational and Technical Inspector for Institutions for the Visually Impaired [jean-marie.cierco@sante.gouv.fr](mailto:jean-marie.cierco@sante.gouv.fr)**

**Françoise MAGNA**

**Head of the Mathematics Subcommittee of the Commission for the Evolution of French Braille  
Certified Mathematics Teacher [fmagna@libertysurf.fr](mailto:fmagna@libertysurf.fr)**

**Please note: printing this document from a computer file may alter the pagination and certain fonts, in particular certain mathematical symbols in the printed version, thereby hindering their proper understanding.**

## Conventions used in this document

In the printed version, Braille characters are represented by a font in which the six-dot cell is always present and only the embossed dots are bold.

**In the Braille version, the character :: (dots 1-2-3-4-5-6) is used to indicate precisely the position of the dots of certain symbols that would otherwise be difficult to recognize.**

**In *this* document, the mathematical modifier is used :: (dot 6) , or**

The code change indicator:  $\hat{\hat{\cdot}}$  (points 6, 3) is only used in examples intended to clarify its use.

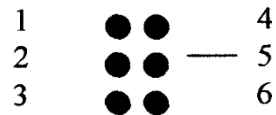
Certain symbols in the printed material are described, in particular when they can have several meanings or when they are very rarely used. Mathematical meanings are separated by semicolons and descriptions are generally written in parentheses.

# INTRODUCTION

## I. Braille writing

A Braille character consists of one to six dots arranged in two columns of three dots. This gives a total of 63 written characters. These dots are usually numbered from top to bottom from one to three for the left column and from four to six for the right column.

The six dots in a Braille cell:



The 63 Braille characters are arranged in a table of 5 rows of 13 characters, as shown in the following table:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
|--|--|--|--|--|--|--|--|--|--|--|--|--|
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  |  |  |  |  |  |

This table calls for the following comments:

a) The last and second-to-last characters in each line have no alphabetical meaning. Some of them are keys, symbols that, when placed before a character, change its meaning.

**b) The first and last lines are constructed in the same way, but the first line never has lower dots, while the last line never has upper dots. The second line is derived from the first by adding the**

point 3: The third is derived from the first by adding points 3 and 6. The fourth is derived from the first by adding point 6.

c) Unlike in print, accented letters and the "linked o e" are separate characters, hence 40 alphabetical characters. Note also the position of the letter w, which here is not the 23rd<sup>me</sup> alphabetical character but the 40th<sup>th</sup>; this letter, which did not exist in the French alphabet at the beginning of the 19th century, was later introduced into Braille at the request of the Anglo-Saxons.

**d) Numbers and capital letters are not represented by a single Braille character.**

The numbers one through nine are obtained by placing the key  $\therefore$  (period 6) before the second through tenth characters of the fourth line, and zero by placing it before the penultimate character of the third line.

A number is written by placing the key  $\therefore$  (point 6) in front of the string of characters that make up that number. When a space separates groups of digits, it is transcribed **by the symbol  $\therefore$  (point 3).**

Capital letters are obtained **by placing the key  $\therefore$  (points 4-6) before the character of the corresponding lowercase letter.**

e) The last line contains punctuation marks: characters 2 to 11, except for the 10<sup>th</sup> character  $\circ$  (points 3-5).

## **II. Transcription of mathematical**

### **CO @ General information**

This document contains the information necessary for transcribing most mathematical formulas. It indicates the correspondence between the printed symbols and the Braille symbols.

When **transcribing** a formula, the following general rule should be used: whenever possible, the signs (numbers, letters, operation signs, etc.) in the formula should be written in the same **order as in the printed text.**

This document also indicates the methods to be used when the above rule is not applicable, for example when the printed formula contains symbols that are not all on the same line (fractions, etc.).

## 2° Formation of symbols braille

Printed symbols are represented by one or more Braille characters. There are single symbols, double symbols, and even a few triple and quadruple symbols.

Double symbols, except for the symbols  $\circ\circ$  (dots 2-5, 2) (implication) and  $\circ\bullet\circ\circ$  (dots 2-5, 2-3-5-6) ("corresponds to"), are always formed in one of the following ways:

- either they are formed of two identical characters,
- or the first of the two characters is one of the following:

$\ddot{\cdot}$  (points 4-6)    $\ddot{\circ}$  (points 4-5)    $\dot{\circ}$  (point 5)    $\dot{\ddot{i}}$  (points 4-5-6)

These four characters or keys are used solely to modify the value of the character that follows them.

**Note:** in very rare cases, the characters  $\dot{\cdot}$  (point 4) and  $\ddot{\cdot}$  (point 6) are also used as keys in contexts where there is no ambiguity with regard to their respective values as exponent indicators and mathematical modifiers

## 3° Processing a mathematical expression

- If a mathematical expression is inserted into a sentence, it is preceded, in full braille as well as contracted braille, the mathematical modifier  $\dot{\cdot}$  (point 6)  $\dot{\cdot}$  (point 6) unless it contains block symbols (see Chapter 8, "Blocks"); in which case the code change indicator is used: v. (points 6, 3).
- If a mathematical expression is "outside the text," the modifier mathematical  $\ddot{\cdot}$  (point 6) or the change indicator change code  $\dot{\cdot}$  (points 6, 3) may be omitted.

NB:

1. Although the systematic use of these symbols is not recommended in Braille "general" braille, it allows, in a scientific text or work, to alert the reader to the "scientific" nature of the writing, as is sometimes done in print by using a different font. In this document, they will only be used in the examples in Chapter 8 "Blocks" and Chapter 16 "Main Units," paragraph 16.3.

"Examples" (see also below, 8° "Use of the abbreviation").

2. In full Braille, the mathematical modifier (dot 6) is not used before writings containing only letters (see Chapter 16 "Main units," paragraph 16.3 "Examples").

## 4 Capital letters in mathematics

In a mathematical expression, each capital letter must be preceded by the key

$\cdot$  (points 4-6) (see in particular examples in Chapter 12, "Superscripts, subscripts, geometry," paragraph 12.1, "Superscripts").



## 5° Breaking up a mathematical expression

The continuation indicator: ° (point 5) placed at the end of a line indicates that a formula continues on the next line.

On break a expression in before a sign of operation, equality, inequality... (see example in Chapter 9 "Indices," paragraph 9.6 "Superscripted or subscripted indices").

However, it is strongly recommended not to break a mathematical expression if it can fit on a single line.

## 6^ Spaces in a formula

In Braille, spaces are not generally used in mathematical formulas. However, they may be inserted to make complex formulas easier to read, but there are no precise rules governing where these spaces should be placed. They should preferably be placed before and/or after an equal sign, inequality sign, operation sign, equivalence sign, etc. (see examples in Chapter 15, "Quantifiers and Implication Signs").

## 7° Punctuation

Punctuation marks are transcribed using the corresponding symbols in Braille.

"general," including ellipsis points (see example in Chapter 9, "Indices," paragraph 9.6, "Superscripted or subscripted indices").

However, in case of ambiguity, the base value indicator :: (points 5-6) should be placed before a closing quotation mark :: (points 2-3-5-6) (so that it is not confused with the equality symbol) or before an exclamation mark :° (points 2-3-5) (so that it is not confused with the "plus" sign).

Regarding semicolon t: (points 2-3), in a formula containing block symbols, it is preceded by the character :: (point 6) so that it is not confused with the end-of-block symbol :: (points 2-3) (see examples in chapter 8, "Blocks").

## 8° Use of the abridged version

The transcription of a mathematical text into Braille can be done using the Abbreviation Extended spelling. However, the following rules should be taken into account:

- a) When isolated lowercase letters or sequences of letters beginning with a lowercase letter are encountered in a text, they may, as in

"general" Braille, be preceded by the base value indicator : (dots 5-6) to avoid confusion with the abbreviation of a word. However, as indicated in 3° "Treatment of a mathematical expression," it is recommended that, for any scientific work or text, they be preceded by the

mathematical modifier :: (point 6) (see exception for lowercase Greek letters, chapter 2 "Different types of letters," paragraph 2.3 "Greek letters"). According to the "general" Braille code, this symbol also has the effect of





# 1. Letters and infinity sign

| configuration of points | meaning  | representation |
|-------------------------|----------|----------------|
| :: (1-6)                | UFI      | 1              |
| :İ (i-2-6)              | two      | 2              |
| :< (1-4-6)              | tFOİS    | 3              |
| :: (i-4-5-6)            | four     | 4              |
| :: (1-5-6)              | CI I9    | 5              |
| °z (1-2-4-6)            | SİX      | 6              |
| :: (1-2-4-5-6)          | seven    | 7              |
| °: (1-2-5-6)            | Î3UÎt    | 8              |
| t. (2-4-6J              | nine     | 9              |
| :: (3-4-s-6)            | zero     | 0              |
| :: :: (<-s, i-4)        | infinity | $\infty$       |

## 2. Different types of the very

### 2.1 letters round

A round letter is transcribed by placing the character  $\circ$  (point 5) before the corresponding letter.

Examples:

| dot configuration                            | meaning            | representation |
|----------------------------------------------|--------------------|----------------|
| $\bullet \bullet$ (points 5, 1-4-s)          | lowercase letter d | $\partial$     |
| $\bullet \bullet \circ$ (points 4-6, 5, 1-j) | uppercase letter c | $\mathcal{C}$  |

### 2.2 "Illuminated" letters

The "enlightened" letters (used for sets of numbers) are transcribed by placing the characters  $::$  (dots 4-6, 4-6) before the corresponding letter.

Examples:

| configuration of dots                                | meaning                          | representation |
|------------------------------------------------------|----------------------------------|----------------|
| $\bullet \bullet \bullet$ (points 4-6, 4-6, 1-3-4-5) | large n (set of natural numbers) |                |
| $\bullet \bullet \circ$ (points 4-6, 4-6, 1-2-3-5)   | large r (set of real numbers)    | $\mathbb{R}$   |

### 2.3 Greek letters

A lowercase Greek letter is transcribed by placing the character:  $\circ$  (points 4-5) before the corresponding Latin letter, and an uppercase Greek letter by placing the characters

$\bullet \bullet$  (points 4-6, 4-5) before the corresponding Latin letter.

Since the construction of lowercase Greek letters is similar to that of certain

currency symbols, they may be preceded by the mathematical modifier  $\circ$  (point 6) to remove any ambiguity. However, this recommendation is optional in the vast majority of cases, as the context does not lend itself to confusion.

### Table showing Greek letters

| Greek letter |                  |                  | associated letter |
|--------------|------------------|------------------|-------------------|
| Name         | lowercase        | uppercase        |                   |
| alpha        | $\alpha$ ' . " ' | A    ' † ' • " ' | a                 |
| beta         | $\beta$ ⋮ ⋮ ⋮    | B    . ? ? .     | b                 |
| gamma        | $\gamma$ ⋮ ⋮ ⋮   | Γ    ⋮ ⋮ ⋮ ⋮     | g                 |
| delta        | $\delta$ ⋮ ⋮ ⋮   | Δ    ⋮ ⋮ ⋮ ⋮     | d                 |
| epsilon      |                  | E    ⋮ ⋮ ⋮ ⋮     | e                 |
| zeta         | $\zeta$ ⋮ ⋮ ⋮    | Z    ⋮ ⋮ ⋮ ⋮     | z                 |
| êta          | $\eta$ ⋮ ⋮ ⋮     | H    > \_ . ? °° | h                 |
| theta        | $\theta$ ⋮ ⋮ ⋮   | Θ    ⋮ ⋮ ⋮ ⋮     | i                 |
| îOtd         | $\iota$ ⋮ ⋮ ⋮    | I    ⋮ ⋮ ⋮ ⋮     | i                 |
| kappa        | ⋮ ⋮ ⋮            | K    ⋮ ⋮ ⋮ ⋮     | k                 |
| lambda       | $\lambda$ ⋮ ⋮ ⋮  | Λ    ⋮ ⋮ ⋮ ⋮     | l                 |
| ITIU         | $\mu$ ⋮ ⋮ ⋮      | M    ⋮ ⋮ ⋮ ⋮     | m                 |
| nu           | ⋮ ⋮ ⋮            | N    ⋮ ⋮ ⋮ ⋮     | n                 |
| ĩTsi OR xĩ   | $\xi$ ⋮ ⋮ ⋮      | ⋮ ⋮ ⋮            | x                 |
| omicron      | $\omicron$ ⋮ ⋮ ⋮ | O    ⋮ ⋮ ⋮ ⋮     | o                 |
|              | ⋮ ⋮ ⋮            | ⋮ ⋮ ⋮            |                   |
| rho          | $\rho$ ¸•        | P    ⋮ ⋮ ⋮ ⋮     | r                 |
| sigma        | $\sigma$ ⋮ ⋮ ⋮   | Σ    ⋮ ⋮ ⋮ ⋮     | s                 |
| tau          | ⋮ ⋮ ⋮            | T    ⋮ ⋮ ⋮ ⋮     | t                 |

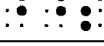
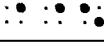
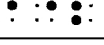
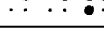
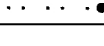
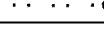
| Greek letter |           |           | related letter |
|--------------|-----------|-----------|----------------|
| noun         | lowercase | uppercase |                |
| upsilon      |           | Υ ⠠⠥⠽⠽⠽⠽  | u              |
| phi          | φ ⠠⠫⠽⠽⠽   | Φ ⠠⠫⠽⠽⠽⠽  | f              |
| khi          | χ ⠠⠭⠽⠽⠽⠽  | Χ ⠠⠭⠽⠽⠽⠽⠽ | q              |
| psi          | ψ ⠠⠫⠽⠽⠽⠽  | Ψ ⠠⠫⠽⠽⠽⠽⠽ | y              |
| omega        | ω ⠠⠽⠽⠽    | Ω ⠠⠽⠽⠽⠽⠽  | w              |

## 2.4 Hebrew letters

A Hebrew letter is transcribed by placing the characters: °:° (dots 4-5, 4-5) in front of the associated letter.

Consonants:

| noun  | Hebrew letter | associated letter |
|-------|---------------|-------------------|
| aleph | א ⠠⠠⠽⠽⠽⠽      | a                 |
| bet   | ב ⠠⠠⠽⠽⠽⠽⠽     | b                 |
| vet   | ב ⠠⠠⠽⠽⠽⠽⠽⠽    | v                 |
| gimel | ג ⠠⠠⠽⠽⠽⠽⠽⠽    | g                 |
| dalet | ד ⠠⠠⠽⠽⠽⠽⠽⠽    | d                 |
| hé    | ה ⠠⠠⠽⠽⠽⠽⠽⠽    | h                 |
| vav   | ו ⠠⠠⠽⠽⠽⠽⠽⠽⠽   | w                 |
| zayin | ז ⠠⠠⠽⠽⠽⠽⠽⠽    | z                 |
| het   | ח ⠠⠠⠽⠽⠽⠽⠽⠽⠽   | x                 |
| tet   | ט ⠠⠠⠽⠽⠽⠽⠽⠽⠽   | t                 |

| f10E          | Hebrew letter                                                                         | associated letter |
|---------------|---------------------------------------------------------------------------------------|-------------------|
| <b>yæ</b>     |      | j                 |
| <b>kaf</b>    | כ    | k                 |
| <b>khaf</b>   | ך    | â                 |
| <b>lamed</b>  | ל    | l                 |
| <b>mem</b>    | מ    | m                 |
| <b>nun</b>    | נ    | n                 |
| <b>samekh</b> | ס    | s                 |
| <b>ayin</b>   | ע    | ë                 |
| <b>pé</b>     | פ    | p                 |
| <b>fé</b>     | ף  | f                 |
| <b>tsadi</b>  | צ  | è                 |
| <b>qof</b>    | ק  | q                 |
| <b>resh</b>   | ר  | r                 |
| <b>shin</b>   | ש  | î                 |
| <b>sin</b>    | ש  | û                 |
| <b>tav</b>    | ת  | t                 |



### 3. operation signs

| point configuration                            | meaning and/or description                                                        | representation |
|------------------------------------------------|-----------------------------------------------------------------------------------|----------------|
| $:\circ$ (points 2-3-5)                        | lgUS                                                                              | $\mp$          |
| $\bullet:\circ$ (points 5, 2-3-s)              | big plus                                                                          | $+$            |
| $:\bullet$ (points 4-6, 2-3-5)                 | plus in a circle                                                                  | $\circ$        |
| $:\bullet$ (points 4-6, 4-6, 2-3-s)            | large plus in a circle                                                            | $\bigcirc$     |
| $:\circ\bullet$ (points 4-5-6, 2-3-s)          | union                                                                             |                |
| $:\circ\bullet$ (points 4-5-6, 4-5-6, 2-3-5)   | great union                                                                       | $\cup$         |
| $\bullet\circ$ (points 4-5, 2-3-5)             | intersection                                                                      |                |
| $\bullet\bullet\circ$ (points 4-5, 4-5, 2-3-5) | great intemection                                                                 | $\cap$         |
| $\dots$ (points 3-6)                           | less                                                                              |                |
| if $\dots$ (points 2-3-5, 3-6)                 | more or less                                                                      | $\pm$          |
| $\dots\hat{\circ}$ (points 3-6, 2-3-s)         | less or more                                                                      |                |
| $\bullet$ (points 3-5)                         | Multiplied by                                                                     | $\times$       |
| $\bullet\bullet$ (points 3-5, 3-5)             | multiplicative point; scalar product (point halfway up the line)                  |                |
| $\circ\bullet$ (points 5, 3-5)                 | asterisk, star; convolution product (multiplication cross crossed out vertically) | $*$            |
| $:\bullet\circ, \circ$ (points S, 5, 3-5)      | large multiplication cross                                                        | $\bigotimes$   |
| $\dots\bullet$ (points 4-6, 3-5)               | tensor product (cross in a circle)                                                | $\otimes$      |
| $\dots\bullet\circ$ (points 4-6, 4-6, 3-5)     | great tensor product (large cross in a circle)                                    | $\bigotimes$   |
| $\circ\circ$ (points 4-5-6, 4-5-6, 3-s)        | double multiplication cross                                                       | $\bigotimes$   |
| $:\circ$ (points 4-5-6, 3-5)                   | factorial (exclamation mark)                                                      |                |
| $\bullet\bullet$ (points 4-5, 3-5)             | vector product; outer product; GCD; conjunction; wedge (sort of inverted v)       |                |
| $\bullet\bullet\bullet$ (points 4-5, 4-5, 3-5) | grand vector product                                                              | $\wedge$       |

| point configuration         | meaning and/or description                                                | representation |
|-----------------------------|---------------------------------------------------------------------------|----------------|
| • • (points 4-5, 2-6)       | <b>PPCPI; inclusive disjunction (sort of v)</b>                           |                |
| : ∴ (points 4-5-6, 3-4-5-6) | <b>product of composition of applications or functions (small circle)</b> | ◦              |
| °° (points 2-5)             | <b>divided by (if placed between two terms: numbers, letters, etc.)</b>   |                |
| ° (points 3-4)              | <b>fraction bar; division slash; quotient of sets</b>                     | $n^f /$        |
| (point 4)                   | exponent indicator, power                                                 |                |

Note: The index is represented by the symbol °. (points 2-6) (see Chapter 9 "Indices," Section 9.2 "Lower Index").

## 4. Equality and inequality signs

| point configuration                   | meaning and/or description                                                                                               | representation |
|---------------------------------------|--------------------------------------------------------------------------------------------------------------------------|----------------|
| ii (points 2-3-5-6)                   | equal                                                                                                                    |                |
| : / ñ (points 4-6, 2-3-5-6)           | different from                                                                                                           | ≠              |
| • :: (points 5, 2-3-5-6)              | slightly different from, approximately equal to (double equivalence sign, equivalence sign underlined once)              | OR = OR 6      |
| ⋮⋮<br>(points 4-5-6, 5, 2-3-5-6)      | double underlined equivalence sign                                                                                       |                |
| ⋯<br>• :: (points 4-5, 2-3-5-6)       | equivalent to; asymptotically equal to                                                                                   |                |
| ⋮⋮<br>(points 4-5-6, 2-3-5-6)         | equivalence underlined twice                                                                                             |                |
| (points 2-3-5-6, 2-3-5-6)             | congruent to; identical to (three horizontal lines superimposed or two horizontal lines superimposed separated by a dot) | - OR -         |
| ⋮⋮⋮<br>(points 4-6, 2-3-5-6, 2-3-5-6) | its negation                                                                                                             | ≠ OU ≠         |
| • ° :: (points 2-5, 2-3-5-6)          | corresponds to (equality sign with a circumflex accent above it)                                                         |                |
| • . ° (points 5, 3-4-5)               | greater than, higher than                                                                                                | >              |
| • . ° (points 4-5, 3-4-5)             | greater than or equal to, higher than or equal to                                                                        | ≥ OU ≥         |
| ⋯<br>• •. (points 5, 1-2-6)           | smaller than, inferior to                                                                                                |                |
| • •. (points 4-5, 1-2-6)              | smaller than or equal to, less than or equal to                                                                          | ≤ OU ≤         |
| ⋮⋮⋮<br>(points 5, 3-4-5, 5, 1-2-6)    | greater than or less than                                                                                                | ><             |
| • : ° . ° (points 5, 5, 3-4-5)        | <b>much greater than; descending order (two "greater than" signs nested within each other)</b>                           |                |

| point configuration                                                                                           | meaning and/or description                                       | representation |
|---------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------|----------------|
| • • • (points 5, 5, 1-2-6)                                                                                    | much less than; ascending order (two nested "less than" signs)   | $\ll$          |
| <br>(points 4-6, 4-6, 3-4-5) | later than, follows (a sort of rounded v, pointing to the right) |                |
| <br>(points 4-6, 4-6, 1-2-6) | before, precedes (a sort of rounded v, pointing to the left)     | $\prec$        |
| (points 4-5, 4-5, 3-4-5)                                                                                      | follows in a broad sense                                         |                |
| <br>(points 4-5, 4-5, 1-2-6) | precedes in a broad sense                                        |                |

## 5. Parentheses, braces, and square brackets

Please note! In mathematical Braille, the symbols representing curly brackets and square brackets are different from those in "general" Braille.

| configuration of dots         | Meaning and/or description                    | Representation |
|-------------------------------|-----------------------------------------------|----------------|
| t. (dots 2-3-6)               | opening parenthesis on a line                 | (...           |
| .: (points 3-5-6)             | closing parenthesis on a line                 | ...)           |
| (points 5, 2-3-6)             | large opening parenthesis on a line (nesting) | (...           |
| (points 5, 3-5-6)             | large closing parenthesis on a line (nesting) | ...)           |
| ⠠⠠⠠<br>(points 4-5, 2-3-6)    | opening parenthesis on several lines          |                |
| ⠠⠠⠠<br>(points 4-5, 3-5-6)    | closing parenthesis on multiple lines         |                |
| (points 4-6, 2-3-6)           | opening brace on one line                     | {...           |
| ⠠⠠⠠<br>(points 4-6, 3-5-6)    | closing brace on one line                     | ...}           |
| (points 4-5-6, 2-3-6)         | opening brace on multiple lines               |                |
| ⠠⠠⠠<br>(points 4-5-6, 3-5-6)  | closing brace on several lines                |                |
| .. (points 1-2-3-5-6)         | opening bracket on one line                   | [...           |
| °° (points 2-3-4-5-6d)        | closing bracket on one line                   | ...]           |
| • ⠠⠠<br>(points 5, 1-2-3-5-s) | large hook opening on a line (interlocking)   | [...           |

| point configuration                                                                                            | meaning and/or description                  | representation     |
|----------------------------------------------------------------------------------------------------------------|---------------------------------------------|--------------------|
| <br>(points 5, 2-3-4-5-6)     | large hook closing on a line (interlocking) | $\dots]$           |
| <br>(stitches 4-5, 1-2-3-5-6) | hook opening on several lines               |                    |
| (stitches 4-5, 2-3-4-5-6)                                                                                      | hook closing on several lines               |                    |
| <br>(points 4-6, 1-2-3-5-6)   | double hook opening on one line             | $\llbracket \dots$ |
| <br>(stitches 4-6, 2-3-4-5-6) | double hook closing on one line             | $\dots \rrbracket$ |

Note: large brackets:  $\circ \therefore$  (points 5, 1-2-3-5-6)  $\circ \therefore$  (points 5, 2-3-4-5-6) frame the large parentheses  $\circ \therefore$  (points 5, 2-3-6)  $\circ \therefore$  {point 5, 3-5-6} which, in turn, frame the brackets  $\therefore$  (points 1-2-3-5-6)  $\therefore$  (points 2-3-4-5-6) which frame the parentheses  $\therefore$  (points 2-3-6)  $\therefore$  (points 3-5-6).

## 6. Enæmblist notations

### 6.1 Sets of numbers

They are denoted using the associated letter preceded by the double symbol  $::$  (points 4-6, 4-6) (see chapter 2 "Different types of letters," paragraph 2.2 "enlightened' letters").

### 6.2 's various

| configuration of points                                                                                   | meaning and/or description                                    | representation |
|-----------------------------------------------------------------------------------------------------------|---------------------------------------------------------------|----------------|
| $\cdot \cdot$ (points 4-6, 1-6)                                                                           | included                                                      |                |
| $\cdot$ (points 4-6, 3-4)                                                                                 | not included                                                  | $\not\subset$  |
| $\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$<br>(points 4-5-6, 4-6, 1-6)        | included in the broad sense                                   |                |
| $\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$<br>(points 4-5-6, 4-6, 3-4)        | not included in the broad sense                               | $\not\supset$  |
| $\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$<br>$\bullet \cdot$ (points 5, 1-6) | containing                                                    |                |
| $\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$<br>$\circ ::$ (points 5, 3-4)      | not containing                                                | $\not\supset$  |
| $\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$<br>$\circ ::$ (points 4-5, 1-6)    | belongs to                                                    |                |
| $\bullet \cdot$ (points 4-5, 3-4)                                                                         | does not belong to                                            | $\notin$       |
| $\begin{smallmatrix} \cdot & \cdot \\ \cdot & \cdot \end{smallmatrix}$<br>(points 4-6, 4-5, 1-6)          | symmetric to $\bullet : \hat{i}$ (points 4-5, 1-6)            |                |
| (points 4-6, 4-5, 3-4)                                                                                    | symmetric to $\circ ::$ (points 4-5, 3-4)                     | $\not\supset$  |
| $\cdot ::$ (points 4-6, 1-4-6)                                                                            | Complementary                                                 | $\complement$  |
| $\circ ::$ (points 4-5, 3-4-5-6)                                                                          | empty set; diameter (capital letter o with a line through it) | $\emptyset$    |





## 7. Divisions, fractions

For a fraction, we use the symbol  $\frac{\cdot}{\cdot}$  (points 3-4).

For a division, we use the symbol  $\div$  (points 2-5).

### Examples:

$\frac{2}{3} = \frac{4}{6}$  two thirds equals four sixths

$\frac{2}{3} = \frac{4}{6}$

$1 \div 2 = 0.5$  one divided by two equals zero point five

$1 \div 2 = 0.5$

**Note:** If one or both terms in a fraction or ratio are complex, blocks must be used (see Chapter 8, "Blocks").

## 8. Blocks

:° (points 5-6) start of block

:: (points 2-3) end of block

**This notation avoids the use of parentheses, which do not appear in printed.**

**The block start symbol alerts the reader that a more or less complex expression is written as a superscript, subscript, etc.**

Blocks are used:

- if linear transcription of the printed text is not possible (fractions, exponents, roots, etc.);
- if the printed text contains spaces that act as parentheses (example:  $\ln x$ );
- if the printed text contains an abbreviation of a word (arg, Card, rot, etc.);
- if their use can make the expression easier to read.

Blocks may be omitted in cases where their omission is not likely to create ambiguity. This is generally true when the expression in superscript, subscript, or under a radical is:

- an integer or decimal number, possibly negative;
- a letter, possibly preceded by the minus sign (including capital letters, Greek letters, round letters, etc.);
- an expression in parentheses.

Reminder : the examples below also illustrate also the use of the mathematical modifier: (point 6) and the code change

indicator: :: (points 6, 3)

**(see Introduction, II. "Transcription of mathematics," 3° "Processing a mathematical expression").**

Examples:

:: :: :: :: :: :: :: :: :: ::

« a plus b » sur a

$$\frac{a+b}{a}$$

. : t : t° : i : :: i : :: ° : ° ::

"a plus b" on "c plus d"

$$\frac{a+b}{c+d}$$

:: \* :: ° : ° : :: ° : ° t : t : :: ° t° : ° : :: ° : °

"a over b" plus "c over d" equals

"ad plus bc" over "bd"

$$\frac{\frac{a}{bc} + \frac{c}{bd}}{bd} = \frac{ad + bc}{bd^2}$$



## 9. Indices

### 9.1 Accents mathematics

Accents, which are placed to the right of letters in print (prime, second, etc.), are represented by apostrophes  $\therefore$  (point 3).

Examples:

$\cdot$  a prime  $ri'$

$\dots \therefore \therefore$  x second  $x''$

$\dots \therefore^\circ \dots \therefore \therefore \therefore^\circ \therefore \therefore \therefore$  a prime x plus a second y plus a third z

$a'x + a''y + a'''z$

### 9.2 Index

It is written after the quantity it affects. It is separated from it by the symbol  $^\circ$ . (points 2-s). If the index is complex, it is placed in a block (see chapter 8 "Blocks").

Examples:

a index p  $\alpha_p$

$\dots -a_0 a$  x index zero  $\alpha_0$

$\dots \therefore^\circ \therefore^\circ \hat{\therefore} \therefore^\circ$  u index "n plus one"  $u_{n+1}$







## 9.6 Superscript indices or subscript

In print, quantities are accompanied by indices placed just above or just below them. These indices are transcribed after the quantity and preceded by double symbols:

- °z (points 2-6, 2-6) subscript  
(points 4, 4) superscript

**Example:**

$$\sum_{i=1}^n u_i = u_1 + u_2 + \dots + u_n$$

sum for i varying from one to n of "u index i" equals "u index one" plus "u index de ux" plus ellipsis plus "u index n"

$$\sum_{i=1}^n u_i = u_1 + u_2 + \dots + u_n$$

Note : one could also write for the first term of equality

- • î: t• t• : t t: tt :: t: • : • : .t :: .. t. t: :: , because the use blocks facilitates the reading and understanding of the expression.



## 10. Radicals

The quantity is written under the radical preceded by the symbol .° (points 3-4-5).

If the exponent of the radical is explicit, it is placed before the symbol  $\sqrt[n]{\phantom{x}}$  (points 3-4-5)

and precede it with the symbol :: (point 4).

If the quantity under the radical is complex, it is placed in a block. Examples:

square root of two

 $\sqrt{2}$ 

• • • • •

fourth root of a

$$\sqrt[4]{a}$$

**Figure 1.** A schematic diagram illustrating the experimental design. The figure shows two rows of stimuli. The top row contains three pairs of stimuli, each pair consisting of a small black dot followed by a larger black dot. The bottom row contains three pairs of stimuli, each pair consisting of a small white dot followed by a larger white dot. Arrows indicate the sequence of stimuli from left to right. Below the stimuli, there are labels indicating the conditions: "Small Dot" and "Large Dot".

sixth root of "(a plus b) cubed" equals square root of "a plus b"

$$^6 \quad (+b) = +b$$

• • • • •

cubic root of "a plus square root of "a plus b"

$$3 + a+b$$

. . . . .  
 . . . . .  
 . . . . .

"fourth root of a" "cubic root of b"

$$\sqrt[4]{a}\sqrt[3]{b}$$

**Note:** in Braille, in writings similar to the one above, blocks must be used to avoid any ambiguity.

## 11. Arrows

| point configuration                                                                                            | meaning and/or description                                                                 | representation                                                                        |
|----------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------|
| <br>(dots 4-5, 1-5-6)         | Arrow pointing up from left to right: crescent                                             |    |
| <br>(points 4-5, 1-2-4-5-6)   | vertical arrow pointing upwards                                                            |    |
| <br>(points 4-5, 2-4-6)       | upward arrow from right to left                                                            |    |
| <br>(points 4-5-6, 1-5-6)     | horizontal arrow to the right                                                              |                                                                                       |
| <br>(points 4-5-6, 1-2-4-5-6) | double horizontal arrow, above from left to right, below from right to left: reversibility |    |
| <br>(points 4-5-6, 2-4-6)   | horizontal arrow to the left                                                               |                                                                                       |
| <br>(points 4-6, 1-5-6)     | downward arrow from left to right: decreasing                                              |  |
| (points 4-6, 1-2-4-5-6)                                                                                        | vertical arrow pointing down                                                               |  |
| <br>(points 4-6, 2-4-6)     | downward arrow from right to left                                                          |  |
| (points 5, 1-5-6)                                                                                              | horizontal arrow from left to right starting with a vertical bar: image is                 |                                                                                       |
| (points 5, 1-2-4-5-6)                                                                                          | horizontal arrow in both directions                                                        |                                                                                       |



## 12. Superscripts, subscripts, geometry notation

In print, a line, angle, arc, arrow, etc. is placed above or below certain letters or groups of letters. In Braille, these are replaced by the following symbols, which are placed before the affected quantities.

### 3.2.1 Superscript signs

| configuration of dots                | meaning and/or description                                                                              | representation |
|--------------------------------------|---------------------------------------------------------------------------------------------------------|----------------|
| •• (points 4, 2-5)                   | arc (half circle convex upward)                                                                         |                |
| • °° (points 4-5, 2-5)               | protruding angle (circumflex accent)                                                                    |                |
| : °° (points 4-6, 2-5)               | vector (horizontal arrow pointing to the right)                                                         |                |
| ⠠⠠⠠⠠⠠⠠ (points 4-6, 2-5, 4, 1-3-4-5) | nth-order tensor                                                                                        |                |
| : (points 4-5-6, 2-5)                | algebraic measure; conjugate of; <b>complement of; class of an element</b> of a set... (horizontal bar) |                |
| :: :: ?• (points 4,4,2-S)            | arc subtended by an interior angle (half-circle convex downward)                                        |                |
| (points 4-5-6, 4-5, 2-5)             | internal angle (inverted circumflex accent)                                                             |                |
| • •••• (points 5, 4-6, 2-5)          | <b>axial vector (circular arrow pointing in the positive direction)</b>                                 |                |
| ⠠⠠⠠ (points 5, 4-5-6, 2-5)           | tilde (inverted and horizontal s, like the equal sign)                                                  |                |
| • •• (points 4-5, 4, z-s)            | negative circular arrow                                                                                 |                |
| ⠠⠠⠠ (points 4-5, 4-5, 2-5)           | inverted T                                                                                              |                |
| ⠠⠠⠠ (points 4-5, 4-6, 2-5)           | dagger (cross)                                                                                          |                |
| ⠠⠠⠠ (points 4-5, 4-5-6, 2-5)         | double horizontal bar                                                                                   |                |

| point configuration                                                                                           | meaning and/or description     | representation                                                                      |
|---------------------------------------------------------------------------------------------------------------|--------------------------------|-------------------------------------------------------------------------------------|
| <br>(points 4-5-6, 4-6, 2-5) | horizontal arrow pointing left |  |

Examples:



**vector v**



**algebraic measure of AB**

$AB$



**arc AB**

$AB$



**angle AOB**

$\widehat{AOB}$



**angle de Ox avec Oy**

$(\overline{6x}, \overline{0})$



angle of vector OA with vector OM

$(\overline{oA}, \overline{OM})$

## 12.2 Signs subscripts

Below a quantity, or some of its indices, there is sometimes an arrow, a line, etc. In Braille, the symbols formed by the identical superscript sign indicated in the table in paragraph 12.1 are placed before the quantity or before the indices concerned, preceded by the character:  $\cdot$  (points 4-6).

Examples:

$\cdot \cdot ^{\circ}$  half-circle convex upward 

$\cdot \cdot \cdot$  circumflex accent

$\cdot \cdot \cdot \cdot \cdot \cdot \cdot$  right arrow 

$\cdot \cdot \cdot \cdot \cdot$  horizontal bar 

$\cdot \cdot \cdot \cdot \cdot$  half-circle convex downward 

$\cdot \cdot \cdot \cdot \cdot \cdot \cdot$  inverted circumflex accent

### 12.3 geometric signs

| configuration of points     | meaning and/or description   | representation |
|-----------------------------|------------------------------|----------------|
| <br>(points 4-5-6, 1-2-5-6) | parallel to                  | //             |
| <br>(points 4-5, 1-2-5-6)   | perpendicular to; anti-thing | ⊥              |

#### Examples:

the line "capital D" is parallel to the line "capital delta"

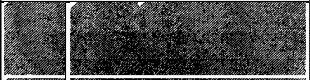
$(D) \parallel (b)$

the line "A B" is perpendicular to the line "A prime B prime"

$(\perp) (A'B')$

## 13. Framing, vertical bars

### 13.1 Notation

| point configuration                                                                                          | meaning and/or description           | representation                                                                       |
|--------------------------------------------------------------------------------------------------------------|--------------------------------------|--------------------------------------------------------------------------------------|
| $::$ (points 1-2-3-4-5-6)                                                                                    | vertical bar on a line               | $  \dots \text{ or } \dots  $                                                        |
| $\circ \circ \circ$<br>(points 4-5-6, 1-2-3-4-5-6)                                                           | vertical bar on several lines        | OR                                                                                   |
| $\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$<br>(points 4-5, 1-2-3-4-5-6)  | double vertical bar on a line        | $\  \dots \text{ ou } \dots \ $                                                      |
| $\begin{smallmatrix} \bullet & \bullet \\ \bullet & \bullet \end{smallmatrix}$<br>(points 4-6, 1-2-3-4-5-6s) | double vertical bar on several lines | $\  \dots \text{ ou } \dots \ $                                                      |
| $\cdot \bullet$ (points 6, 3-4-5)                                                                            | line break in a table                |   |
| $\bullet \bullet$ (points 5,2)                                                                               | empty cell in a table                |  |

### 13.2 Absolute value, modulus, norm...

Examples:

$|x|$  absolute value of "x index zero"  $| \circ |$

$|a + ib|$  module of "a plus ib"  $|a + ib|$

$\|k\vec{u}\|$

norm of "k vector u" equals "absolute value of k" "norm of vector u"

$$\|k\vec{u}\| = |k| \|\vec{u}\|$$

$f|_E$  f restricted to the set E  $f|_E$



### 13.3 s and matrix determinants

In Braille, determinants and matrices are transcribed, as far as possible, as in print. In this case:

- vertical bars or parentheses are transcribed as symbols on a single line;
- **To symbolize an empty box, the symbol :° °: (points 5, 2) (empty box) can be omitted, leaving the corresponding space.**

However, if the expressions or terms in the determinants or matrices are disproportionate, linear notation can be used with the symbol :. ° (points 6, 3-4-5) (line break in a table). In this case, the vertical bars or parentheses are transcribed as symbols on several lines at the beginning and end of the determinant or matrix.

#### Examples:

$$\begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = 3$$

I equals determinant "1, 0; 2, 3" equals 3 
$$\begin{vmatrix} 1 & 0 \\ 2 & 3 \end{vmatrix} = 3$$

$$\begin{vmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \end{vmatrix} = 1 \cdot \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 0 \cdot \begin{vmatrix} 1 & 6 \\ 3 & 9 \end{vmatrix} + 2 \cdot \begin{vmatrix} 1 & 4 \\ 3 & 8 \end{vmatrix} = 1 \cdot (4 \cdot 6 - 5 \cdot 5) - 0 + 2 \cdot (1 \cdot 8 - 3 \cdot 3) = 1 \cdot (24 - 25) + 2 \cdot (8 - 9) = -1 - 2 = -3$$

or in linear form

$$\begin{vmatrix} 1 & 0 & 2 \\ 3 & 4 & 5 \end{vmatrix} = 1 \cdot \begin{vmatrix} 4 & 5 \\ 5 & 6 \end{vmatrix} - 0 \cdot \begin{vmatrix} 1 & 6 \\ 3 & 9 \end{vmatrix} + 2 \cdot \begin{vmatrix} 1 & 4 \\ 3 & 8 \end{vmatrix} = 1 \cdot (4 \cdot 6 - 5 \cdot 5) - 0 + 2 \cdot (1 \cdot 8 - 3 \cdot 3) = -1 - 2 = -3$$

J equals the matrix "1, empty cell, empty cell, 4; 2, 3, 5, 6; -7, 8, empty cell, 9."

$$J = \begin{pmatrix} 1 & & & 4 \\ 2 & 3 & 5 & 6 \\ -7 & 8 & & 9 \end{pmatrix}$$

$$K = \frac{1}{\frac{x+3}{+5}} \frac{x^2+3x-9}{6^3-x^2+1} g(x)$$

K equals the matrix "1, quotient of 'x squared plus 3x minus 9' by '6x cubed minus x squared plus 1'; quotient of 'x plus 3' by 'y plus 5', 'g of x'"

$$K = \frac{1}{\frac{x+3}{+5}} \frac{x^2+3x-9}{6^3-x^2+1} g(x)$$

### 13.4 System

In Braille, the opening brace symbol is placed at the beginning of a system written over several lines :: :: (dots 4-5-6, 2-3-6) before the first mathematical expression (equation, inequality, etc.).

Example:

$$\begin{cases} 2x + 3y = 5 \\ 3x - 7y = 8 \end{cases}$$

or in linear

$$\begin{cases} 2x + 3y = 5 \\ 3x - 7y = 8 \end{cases}$$

The system of equations "two x plus three y equals five" and "three x minus seven y equals eight"

$$\begin{cases} 2x + 3y = 5 \\ 3x - 7y = 8 \end{cases}$$

Note: to make such expressions easier to read, it is more common to write them on several lines, as in print.

## 14. Functions

### 14.1 Notation

| point configuration                      | meaning and/or description                                                                                    | representation |
|------------------------------------------|---------------------------------------------------------------------------------------------------------------|----------------|
| $\hat{\cdot}$<br>(points 4-5-6, 3-4-5-6) | product of function composition (small round)                                                                 |                |
| (points 5, 1-4-5)                        | partial derivative (letter "d")                                                                               | $\partial$     |
| $\Delta$<br>(points 4-6, 4-5, 1-4-5)     | laplacian (large delta)                                                                                       | $\Delta$       |
| (points 4-5-6, 1-4-5-6)                  | d'Alembertian (small square)                                                                                  |                |
| $\nabla$<br>(points 4-6, 1-4-5-6)        | absolute differential, covariant, nabla<br>(equilateral triangle pointing downwards,<br>large inverted delta) | $\nabla$       |

#### Examples:

$\hat{\cdot}$

f of x

$f(x)$

" g prime of x equals "three x plus four"

g prime of x equals "three x plus four"

$g'(x) = 3x + 4$

$f \circ g$

"f round g" of x equals f of "g of x"

$f \circ g(x) = f[g(x)]$

$\frac{\partial}{\partial x}$

"d round" "f of x and y" on "d round" x equals "five xy minus seven x"

$\frac{\partial}{\partial x} - 5 - 7x$



### 14.3 Sign tables, variation tables

When transcribing tables of signs and tables of variation, the following observations should be taken into account:

- the horizontal separator line may be omitted (see example 4),
- the "double bar" in a table is transcribed using the symbol :: (points 1-2-3-4-5-6) and not by the symbol :° :t (points 4-5, 1-2-3-4-5-6),
- In cases where all the information in a line of the original table cannot be written on a single line in braille, several methods are suggested (see examples 3 and 4).

Example 1

|        |           |               |           |
|--------|-----------|---------------|-----------|
| $x$    | $-\infty$ | $\frac{3}{2}$ | $+\infty$ |
| $2x-3$ |           |               |           |
|        | -         | 0             | +         |

Braille transcription of Example 1:

Example 2

|         |           |            |            |
|---------|-----------|------------|------------|
| $x$     | $-\infty$ | $7$        | $+\infty$  |
| $f'(x)$ |           |            |            |
|         | +         | 0          | -          |
| $f(x)$  | $-\infty$ | $\nearrow$ | $\searrow$ |
|         |           | -1         | $-\infty$  |

Braille transcription of Example 2:

Example 3

|         |           |            |      |            |                             |            |           |
|---------|-----------|------------|------|------------|-----------------------------|------------|-----------|
| $x$     | $-\infty$ |            | $-4$ |            | $\frac{2}{3}$               |            | $+\infty$ |
| $f'(x)$ |           | $-$        | $0$  | $+$        | $\parallel$                 | $-$        |           |
| $f(x)$  | $+\infty$ | $\searrow$ | $1$  | $\nearrow$ | $+\infty \parallel +\infty$ | $\searrow$ | $5$       |

|         |           |            |      |            |                             |            |           |
|---------|-----------|------------|------|------------|-----------------------------|------------|-----------|
| $x$     | $-\infty$ |            | $-4$ |            | $\frac{2}{3}$               |            | $+\infty$ |
| $f'(x)$ |           | $-$        | $0$  | $+$        | $\parallel$                 | $-$        |           |
| $f(x)$  | $+\infty$ | $\searrow$ | $1$  | $\nearrow$ | $+\infty \parallel +\infty$ | $\searrow$ | $5$       |

or, by splitting the table

|         |           |            |      |            |                             |            |           |
|---------|-----------|------------|------|------------|-----------------------------|------------|-----------|
| $x$     | $-\infty$ |            | $-4$ |            | $\frac{2}{3}$               |            | $+\infty$ |
| $f'(x)$ |           | $-$        | $0$  | $+$        | $\parallel$                 | $-$        |           |
| $f(x)$  | $+\infty$ | $\searrow$ | $1$  | $\nearrow$ | $+\infty \parallel +\infty$ | $\searrow$ | $5$       |

### Example 4

$$f(x) \Big|_{-\infty}^{\infty} \nearrow \frac{1 - \sqrt{\frac{5}{3}}}{7} \searrow -\infty \parallel +\infty \searrow \frac{2}{3}$$

The figure consists of 10 small diagrams arranged in a single row, each showing a pattern of black dots on a grid. The patterns evolve from left to right. The first diagram has 5 dots. The second has 6 dots. The third has 7 dots. The fourth has 8 dots. The fifth has 9 dots. The sixth has 10 dots. The seventh has 11 dots. The eighth has 12 dots. The ninth has 13 dots. The tenth has 14 dots. The patterns are roughly rectangular, with some internal structure that becomes more complex over time.

Figure 1 shows five 3x3 dot patterns labeled (a) through (e). Each pattern consists of black dots on a white background. Pattern (a) has 8 dots, pattern (b) has 6 dots, pattern (c) has 6 dots, pattern (d) has 6 dots, and pattern (e) has 8 dots.

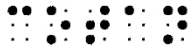
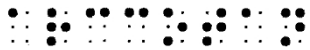






## 14.6 Trigonometry

The same notation is used as in print, especially in documents that are widely distributed. If, in print, expressions in a trigonometric formula are not written in parentheses, blocks are used in Braille.

| point configuration                                                                                                                              | meaning and/or description | representation |
|--------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------|----------------|
| <br>(dots 2-3-4, 2-4, 1-3-4-5)                                  | sine                       | sin            |
| <br>(points 1-4, 1-3-5, 2-3-4)                                  | cosine                     | cos            |
| (points 2-3-4-5, 1, 1-3-4-5)                                                                                                                     | tangent                    | tan            |
| <br>(points 1-4, 1-3-5, 2-3-4-s, i, 1-3-4-5)                    | cotangent                  | cotan          |
| (points 1, 1-2-3-5, 1-4, 2-3-4, 2-4, 1-3-4-5)                                                                                                    | arc sine                   | arcsin         |
| (points 1, 1-2-3-5, 1-4, 1-4, 1-3-5, 2-3-4)                                                                                                      | arc cosine                 | arccos         |
| <br>(points 1, 1-2-3-5, 1-4, 2-3-4-5, 1, 1-3-4-5)             | arc tangent                | arctan         |
| <br>(points 1, 1-2-3-5, 1-4, 1-4, 1-3-5, 2-3-4-5, 1, 1-3-4-5) | arc cotangent              | arccotan       |



## 14.7 Hyperbolic lines

The same notation is used as in print.

**If, in print, the expressions in a hyperbolic formula are not written in parentheses, blocks are used in Braille.**

Example:

$$\cosh^2 x - \sinh^2 x = 1$$

hyperbolic cosine squared minus hyperbolic sine squared equals one

$$\cosh^2 x - \sinh^2 x = 1$$

## 15. Quantificators and signs of implication

| point configuration                 | signification and/or description                              | representation |
|-------------------------------------|---------------------------------------------------------------|----------------|
| t ∴ (points 4-5-6, 1-6)             | existential quantifier: there exists                          | ∃              |
| (points 4-6, 4-5-6, 1-6)            | its negation                                                  | ∄              |
| ∴ ∴ ∴<br>(points 4-5-6, 1-6, 2-3-5) | there exists a single                                         | ∃!             |
| ∴ ∴ (points 4-5-6, 3-4)             | universal quantifier: whatever, for all                       | ∀              |
| ∴ ∴<br>(points 4-6, 4-5-6, 3-4)     | its negation                                                  | ∄              |
| ∴ (points 2-5)                      | such as (represented by a colon OR a slash or a vertical bar) | : OR / OR      |
| " ∴ (points 2-5, 2)                 | right implication: implies, leads to right                    |                |
| ∴ ∴<br>(points 4-6, 2-5, 2)         | its negation                                                  | ∄              |
| ∴ ∴ (points 5, 2-5)                 | implication on the left: is implied by, entails on the left   | ⇐              |
| ∴ ∴<br>(points 4-6, 5, 2-5)         | its negation                                                  | ∄              |
| ∴ ∴ ∴<br>(points 5, 2-5, 2)         | logical equivalence: is equivalent to                         |                |
| ∴ ∴ ∴ ∴<br>(points 4-6, 5, 2-5, 2)  | its negation                                                  | ≠              |



## 16. Main units

### 16.1 General

- 1) **The same notation is used as in print. As in print, the numerical part should generally be separated from the unit by a space.**
- 2) The symbols for units are based on those used in print. In abbreviated Braille, in accordance with the guidelines set out in the introduction, II. 8° "Use of abbreviations," in a scientific document or work, units represented by isolated lowercase letters or sequences of letters beginning with a lowercase letter are preceded by the modifier  
  
mathematical modifier :: (point 6).
- 3) Where possible, in accordance with typographical standards for printed material, a number and its unit are written on the same line.

## 16.2 Notation

### a) Decimal multiples and submultiples

⠠⠠⠠⠠ péta P ( $10^{15}$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ téra T ( $10^{12}$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ giga G ( $10^9$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ mega M ( $10^6$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ hectokilo hk ( $10^3$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ myria ma ( $10^4$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ kilo k ( $10^3$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ hecto h ( $10^2$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ deca da ( $10^1$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ déci d ( $10^{-1}$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

centi c ( $10^{-2}$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ milli m ( $10^{-3}$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ micro  $\mu$  ( $10^{-6}$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ nano n ( $10^{-9}$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ pico p ( $10^{-12}$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠

⠠⠠⠠⠠ femto f ( $10^{-15}$ ) ⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠



## b) Length

|              |            |                                                 |
|--------------|------------|-------------------------------------------------|
| ⋮⋮⋮⋮⋮        | myriameter | mam                                             |
| • e•         |            |                                                 |
| ⋮            | kilometer  | km                                              |
| • • •<br>••  | hectometer | hm                                              |
| • • •<br>• • | of cameter | dam                                             |
| ⋮            | meter      | m                                               |
| • • •<br>• • | decimeter  | dm                                              |
| ⋮            | centimeter | cm                                              |
| ⋮            |            |                                                 |
| ⋮ ⋮          | millimeter | mm                                              |
| • • •<br>• • | micrometer | μm                                              |
| ⋮⋮⋮⋮⋮ ⋮⋮     | angström   | Å (capital letter with a small circle above it) |

## c) Area or surface area

|                    |                      |                                    |
|--------------------|----------------------|------------------------------------|
| ⋮ ⋮ ⋮              | square kilometer     | km <sup>2</sup>                    |
| • • ⋮ ⋮            | square hectometer    | hm <sup>2</sup>                    |
| • • • • • •<br>• • | square<br>decameter  | dam <sup>2</sup><br>m <sup>2</sup> |
| ⋮ ⋮ ⋮              | square meter         | dm <sup>2</sup>                    |
| • G • • • t        | square decameter     | cm <sup>2</sup>                    |
| • • • • •<br>⋮ ⋮ ⋮ | square<br>centimeter | mm <sup>2</sup>                    |
| • ⋮ ⋮              | square millimeter    |                                    |



## f} Capacity

|         |            |     |
|---------|------------|-----|
| •••••   | kiloliter  | kL  |
| “ . . . | hectoliter | hL  |
| “ . . . | 'decaliter | daL |
| •••••   | litre      | L   |
| “ . . . | 'deciliter | dL  |
| • . . . | centiliter | cL  |
| •••••   | millilitre | mL  |

## g) Mass

|         |               |     |
|---------|---------------|-----|
|         | ton           | t   |
|         | hundredweight | q   |
| • <a    | kilogram      | kg  |
| ••      | hectogram     | hg  |
| “ . . . | 'decagram     | dag |
| •       | gram          | g   |
| “ . . . | 'decigram     | dg  |
| ••      | centigram     | cg  |
| ••      | milligram     | mq  |

## h) Currency

|       |           |   |
|-------|-----------|---|
| • •   | euro      | € |
| .e    | euro cent | c |
| • • • |           |   |
| › °:  | franc     | F |

**Note:** in Braille, as the construction of the euro symbol (€) is similar to that of the lowercase Greek letter epsilon (c), the mathematical modifier can be used.

. (point 6) before the Greek letter to avoid any ambiguity. However, this recommendation is optional in the vast majority of cases, as the context does not lend itself to confusion.

## i) Calorimetry

|           |            |     |
|-----------|------------|-----|
| :° °°     | therm      | th  |
| i i° °°   | millitherm | mth |
| • . :° °° | microtherm | pth |
| “ ” , “ ” | calorie    | cal |

## j} Work, energy

|             |                |     |
|-------------|----------------|-----|
| ” : °°      | joule          | J   |
| . . . • •   | kilojoule      | kJ  |
| › • ^ • •   | watt hour      | Wh  |
| . °° .:     | kilogram-meter | kgm |
| : ” t :: .. | electron volt  | eV  |

### **k} Power**

$\text{::} \cdot \text{::} \cdot \text{::}$       kilowatt      **kW**

$\cdot \cdot \text{::}$       **watt**      **W**

horsepower      **hp**

### **l) Force**

$\text{::} \cdot \cdot$       newton      **N**

$\text{::}^* \cdot \cdot$       sten      **sn**

### **m) Pressure**

$\text{::} \cdot \text{::} \cdot \text{::}$       pascal      **Pa**

$\text{::} \cdot \text{::} \cdot \text{::}$       bar      **bar**

$\cdot \cdot$       millibar      **mb**

$\text{::} \cdot$   
 $\text{::} \cdot \text{::}$       piezo      **pz**

## n) Electricity, magnetism

|             |         |          |
|-------------|---------|----------|
| $\text{A}$  | ampere  | A        |
| $\text{C}$  | coulomb | C        |
| $\text{V}$  | volt    | V        |
| $\Omega$    | ohm     | $\Omega$ |
| $\text{F}$  | farad   | F        |
| $\text{H}$  | henry   | H        |
| $\text{Wb}$ | weber   | Wb       |
| $\text{T}$  | tesla   | T        |
| $\text{Mx}$ | maxwell | Mx       |

## o) Optics

|             |         |    |
|-------------|---------|----|
| $\text{cd}$ | candela | cd |
| $\text{lm}$ | lumen   | lm |
| $\text{lx}$ | lux     | lx |
| $\text{ph}$ | phot    | ph |
| $\text{b}$  | diopter | b  |

p) Radioactivity

$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \cdot & \cdot & \cdot \end{smallmatrix}$  becquerel Bq

$\therefore \text{''}$  curie Ci

$\therefore^{\circ\circ} \therefore$  gray Gy

$\therefore \text{''''} \text{''}$  rad rad

$\therefore \therefore^{\circ}$  rontgen R

$\therefore \text{t} \therefore$  sievert Sv

q) Angle and arc

$\bullet \therefore^{\circ}$  degree  $^{\circ}$

$\therefore$  minute

$\rangle \rangle$  second  $\text{''}$

$\therefore \therefore$  grade gr

$\begin{smallmatrix} \bullet & \bullet \\ \cdot & \cdot \end{smallmatrix}$   
 $\hat{\bullet} \bullet$  radian rd

$\text{i} \therefore$  steradian sr

#### r) Time, frequency

|             |        |     |
|-------------|--------|-----|
| •           | hour   | h   |
| ⋮ ⋮ ⋮       | minute | min |
| <i>t</i> '' | second | s   |
| . : : : :   | henz   | Hz  |

#### s) Acoustic

|            |         |    |
|------------|---------|----|
| ⋮ ⋮        | bel     | B  |
| . ° .. ° : | decibel | dB |

#### t) Temperature

|               |                |    |
|---------------|----------------|----|
| • . ° : : : : | degree Celsius | °C |
| < . :         | kelvin         | K  |



## 16.3 Examples

### a) In full Braille

$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$  five point three kilometers 5.3 km

$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$  three point forty euros €3.40

$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$  one point five kilowatts 1.5 kW

$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$  two hundred twenty volts 220 V

$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$  sixty-seven square kilometers 67 km<sup>2</sup>

$\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$  thirty-seven degrees (angle, arc, or temperature) 37°

forty-five degrees twenty-five minutes twelve seconds (angle or arc) 45° 25' 12"

$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$  zero degrees Celsius 0 °C

$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$  fifty kilometers per hour 50 km/h

three point four meters per second minus one 3.4 m/s<sup>-1</sup>

$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$

two hours thirty-five minutes fifteen seconds 2 hours 35 minutes 15 seconds

$\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$   $\begin{smallmatrix} \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet \end{smallmatrix}$  seven liters 7 L







## SUMMARY TABLE

### I. Value of keys before a letter

|                      |                                                     |
|----------------------|-----------------------------------------------------|
| • (points 4-6)       | regular uppercase letter                            |
| • (points 4-5-6)     | italic; bold; underlined                            |
| • (points 4-5)       | lowercase Greek letter                              |
| •° (points 4-6, 4-5) | uppercase Greek letter                              |
| • (point 5)          | lowercase round letter                              |
| •° (points 4-6, 5)   | uppercase round letter                              |
| •• (points 4-6, 4-6) | "illuminated" letter (referring to sets of numbers) |
| •• (points 4-5, 4-5) | Hebrew letter                                       |

### II. Symbols obtained by combining keys and numbers

|                                  |                                     |    |
|----------------------------------|-------------------------------------|----|
| • (point 6, 1-6)                 | isolated number                     | 1  |
| •° (points 5, 1-6)               | containing                          |    |
| •• (points 4-5, 1-6)             | belongs to                          | e  |
| ••°• (points 4-6, 4-5, 1-6)      | symmetrical to •• (points 4-5, 1-6) |    |
| •• (points 4-6, 1-6)             | included                            |    |
| •••• (points 4-5-6, 4-6, 1-6)    | included in the broad sense         |    |
| •• (points 4-5-6, 1-6)           | there exists                        | ∃  |
| •••• (points 4-6, 4-5-6, 1-6)    | its negation                        | ∄  |
| ••••° (points 4-5-6, 1-6, 2-3-5) | there exists a unique               | ∃! |

|                                                                                                              |                                      |               |
|--------------------------------------------------------------------------------------------------------------|--------------------------------------|---------------|
| $\text{ : i }^{\circ}\text{ i}$ (points 6, 1-2-6)                                                            | <b>number two</b>                    | <b>2</b>      |
| $\text{ . }^{\circ}\text{ }^{\circ}$<br><b>• • (points 5, 1-2-6)</b>                                         | <b>isolated lower</b>                | <b>&lt;</b>   |
| $\text{ }^{\circ}\text{ }^{\circ}\text{ }^{\circ}$ (points 5, 5, 1-2-6)                                      | <b>than</b>                          | <b>&lt;</b>   |
| <b>• • (points 4-5, 1-2-6)</b>                                                                               | <b>much lower than</b>               | <b>1 OR 1</b> |
|                                                                                                              | less than or equal to                |               |
| $\text{ : }^{\circ}\text{ : }^{\circ}\text{ }^{\circ}$ (points 4-6, 4-6, 1-2-6)                              | <b>prior to</b>                      | <b>&lt;</b>   |
| $\text{ }^{\circ}\text{ : }^{\circ}\text{ }^{\circ}$ (points 4-5, 4-5, 1-2-6)                                | preceding in the broad sense         | <b>&lt;</b>   |
| <b> . . (points 6, 1-4-6)</b>                                                                                | <b>number three isolated</b>         | <b>3</b>      |
| $\text{ . }^{\circ}\text{ }^{\circ}$ (points 4-6, 1-4-6)                                                     | <b>complementary</b>                 | <b>C</b>      |
| $\text{ . i }^{\circ}\text{ }^{\circ}$ (points 6, 1-4-5-6)                                                   | <b>isolated number four</b>          | <b>4</b>      |
| <b> . :^{\circ} (points 4-6, 1-4-5-6)</b>                                                                    | <b>absolute differential of</b>      |               |
| $\text{ }^{\circ}\text{ I}$ (points 4-5-6, 1-4-5-6)                                                          | <b>Alembertian</b>                   |               |
| $\text{ }^{\circ}\text{ }^{\circ}\text{ }^{\circ}$<br>$\text{ . }^{\circ}\text{ }^{\circ}$ (points 6, 1-5-6) | <b>isolated digit five</b>           | <b>5</b>      |
| $\text{ }^{\circ}\text{ }^{\circ}\text{ }^{\circ}$<br><b>• : (points 5, 1-5-6)</b>                           | a forimage <b>ascending</b>          | <b>↗</b>      |
| $\text{ }^{\circ}\text{ }^{\circ}\text{ }^{\circ}$<br><b>• : (points 4-5, 1-5-6)</b>                         | <b>descending</b>                    | <b>↘</b>      |
| <b> . :^{\circ} (points 4-6, 1-5-6)</b>                                                                      | <b>horizontal arrow to the right</b> |               |
| $\text{ }^{\circ}\text{ }^{\circ}\text{ }^{\circ}$ (points 4-5-6, 1-5-6)                                     | <b>isolated digit six</b>            | <b>6</b>      |
| <b> . • (points 6, 1-2-4-6)</b>                                                                              |                                      |               |

|                                                |                                     |                      |
|------------------------------------------------|-------------------------------------|----------------------|
| $\cdot^{\circ\circ}$ (points 6, 1-2-4-5-6)     | number seven isolated               | 7                    |
| $\bullet^{\circ}$ (points 5, 1-2-4-5-6)        | horizontal arrow in both directions | $\leftrightarrow$    |
| $\bullet^{\circ\circ}$ (points 4-5, 1-2-4-5-6) | vertical arrow pointing up          | $\uparrow$           |
| $\bullet^{\circ\circ}$ (points 4-6, 1-2-4-5-6) | vertical arrow pointing down        | $\downarrow$         |
| $::^{\circ}$ (points 4-5-6, 1-2-4-5-6)         | reversibility                       | $\rightleftharpoons$ |
| $\cdot^{\circ}$ (points 6, 1-2-5-6)            | isolated figure eight               | 8                    |
| $:^{\circ}$ (points 4-5-6, 1-2-5-6)            | parallel to                         | //                   |
| $:^{\circ}$ (points 4-5, 1-2-5-6)              | perpendicular to; anti-trick        | $\perp$              |
| $\cdot^{\circ}$ (points 6, 2-4-6)              | isolated number nine                | 9                    |
| $\bullet^{\circ}$ (points 4-5, 2-4-6)          | upward arrow from right to left     | $\nwarrow$           |
| $\cdot^{\circ}$ (points 4-6, 2-4-6)            | downward arrow from right to left   | $\swarrow$           |
| $:^{\circ}$ (points 4-5-6, 2-4-6)              | horizontal arrow to the left        | $\dashleftarrow$     |
| $\cdot^{\circ}$ (points 6, 3-4-5-6)            | isolated zero                       | 0                    |
| $\bullet^{\circ}$ (points 4-5, 3-4-5-6)        | empty assembly; diameter            | $\emptyset$          |
| $:^{\circ}$ (points 4-5-6, 3-4-5-6)            | product composition                 | $\circ$              |

### III. Symbols obtained by combining keys and characters other than letters and numbers

|                                  |                                                |                       |
|----------------------------------|------------------------------------------------|-----------------------|
| (pointR)                         | exponent indicator, superscript                |                       |
| • e (points 4, 4)                | superscript                                    | ,                     |
| ''' (points 6, 4)                | superscript on the left in chemistry           | ‘Q                    |
| . ° (points 3-4)                 | fraction bar; division slash; quotient of sets |                       |
|                                  | not containing                                 | — or /                |
| • ∴ (points 5, 3-4)              | does not belong to                             | ∄                     |
| • ∙ (points 4-5, 3-4)            |                                                | ∉                     |
| ∴ ∴ ∴ (points 4-6, 4-5, 3-4)     | symmetrical to                                 | ∴ ∴ (points 4-6, 3-4) |
|                                  |                                                | y                     |
| ∴ ∴ (points 4-6, 3-4)            | not included                                   | ⊄                     |
| ∴ ∴ ∴ (points 4-5-6, 4-6, 3-4)   | not included in the broad sense                | ⊈                     |
| ∴ ∴ ∴ (points 4-5-6, 3-4)        | whatever, for all                              | ∀                     |
| ∴ ∴ ∴ (points 4-6, 4-5-6, 3-4)   | its negation                                   | ∄                     |
| • ∴ (points 3-4-5)               | root                                           | √                     |
| ∴ ∴ ∴ (points 4, 1-3-4-5, 3-4-5) | nth root                                       | <sup>n</sup> √        |
| ∴ ∴ ∴ (points 6, 3-4-5)          | line break in a table                          |                       |
| • ∴ (points 5, 3-4-5)            | greater than                                   | >                     |



|                                       |                                            |        |
|---------------------------------------|--------------------------------------------|--------|
| • . . . . (points 5, 3-4-5, 5, 1-2-6) | greater than or less than                  |        |
| • ∴ ∴ (points 5, 5, 3-4-5)            | much greater than                          | ≫      |
| • ∘ (points 4-5, 3-4-5)               | greater than or equal to                   | ≥ ou ≥ |
| ∴ ∴ ∘ (points 4-6, 4-6, 3-4-5)        | after                                      | Υ      |
| • • ∘ (points 4-5, 4-5, 3-4-5)        | follows in the broad sense                 | Υ      |
| • (point 2)                           | comma                                      |        |
| ∴ ∘ (point 5)                         | <b>continuation indicator empty box</b>    |        |
| • ∘ (points 5, 2)                     | <b>in a table</b>                          |        |
| ∴ (point 3)                           | premium;                                   |        |
|                                       | <b>space separating digits in a number</b> |        |
| › › (points 3, 3)                     | second                                     | "      |
| . ‹ . (points 3, 3, 3)                | terce                                      | "      |
| . (point 6)                           | mathematical modifier                      |        |
| . (points 6, 3)                       | code change indicator                      |        |
| ∴ (points 5-6)                        | <b>start of block</b>                      |        |
| ∴ (points 2-3)                        | end of block                               |        |

|                                                          |                                     |                   |
|----------------------------------------------------------|-------------------------------------|-------------------|
| $\bullet^\circ$ (points 2-5)                             | <b>colon; divided by; such that</b> |                   |
| $\bullet^\circ$ : (points 2-5, 2)                        | right implication:                  |                   |
|                                                          | imptique, entails on the right      | $\Rightarrow$     |
| $\cdot^\circ^\circ$ : (points 4-6, 2-5, 2)               | its negation                        | $\nRightarrow$    |
| $\bullet \bullet$ (points 5, 2-5)                        | implication on the left:            |                   |
|                                                          | is implied by, entails on the left  |                   |
| $\cdot^\circ^\circ$ (points 4-6, 5, 2-5)                 | <b>its negation</b>                 | $\nRightarrow$    |
| $\cdot \cdot \cdot \cdot \cdot$ (points 5.2-5.2)         | logical equivalence: equivalent to  | $\Leftrightarrow$ |
| $\therefore^\circ^\circ^\circ$ : (points 4-6, 5, 2-5, 2) | its negation                        | $\nRightarrow$    |

## Superscript signs

|                                                               |                                          |                      |
|---------------------------------------------------------------|------------------------------------------|----------------------|
| $\cdot \cdot$ (points 4, 2-5)                                 | arc                                      | '                    |
| $\bullet \bullet$ (points 4-5, 2-5)                           | angle                                    | $\hat{\square}$      |
| $\cdot^\circ^\circ$ (points 4-6, 2-5)                         | <b>vector</b>                            | $\vec{\square}$      |
| $\cdot \bullet * \cdot^\circ$ (points 4-6, 2-5, 4, 1-3-4-5)   | <b>tensor of order n</b>                 | $\mathcal{Q}'$       |
| $:\circ^\circ$ (points 4-5-6, 2-5)                            | horizontal bar                           | $\overline{\square}$ |
| $\cdot \cdot \cdot \cdot \cdot$ (points 4, 4, 2-5)            | arc subtended by an obtuse angle         |                      |
| $:\circ^\circ^\circ$ (points 4-5-6, 4-5, 2-5)                 | interior angle                           | $\square$            |
| $\bullet \cdot^\circ^\circ$ (points 5, 4-6, 2-5)              | axial vector,<br>positive circular arrow | $\vec{\square}$      |
| $^\circ \cdot \hat{\cdot}^\circ^\circ$ (points 5, 4-5-6, 2-5) | tilde                                    | $\tilde{\square}$    |

|                                 |                                |                                                                                     |
|---------------------------------|--------------------------------|-------------------------------------------------------------------------------------|
| • •• (points 4-5, 4, 2-5)       | negative circular arrow        |  |
| • •• (points 4-5, 4-5, 2-5)     | <b>upside-down</b>             |  |
| :• :• °° (points 4-5, 4-6, 2-5) | dagger                         |  |
| • :•• (points 4-5, 4-5-6, 2-5)  | <b>double horizontal bar</b>   |  |
| ↑ •• (points 4-5-6, 4-6, 2-5)   | horizontal arrow pointing left |                                                                                     |

## Subscript signs

|                                                                           |                                    |                                                                                       |
|---------------------------------------------------------------------------|------------------------------------|---------------------------------------------------------------------------------------|
| • •• (points 4-6, 4, 2-5)                                                 | <b>half-circle convex upward</b>   |    |
| • •• (points 4-6, 4-5, 2-5)                                               | <b>circumflex accent</b>           |    |
| • •• (points 4-6, 4-6, 2-5)                                               | <b>right-pointing arrow</b>        |                                                                                       |
| • :•• (points 4-6, 4-5-6, 2-5)                                            | <b>horizontal bar</b>              |  |
| • •• (points 4-6, 4, 4, 2-5)                                              | <b>half-circle convex downward</b> |                                                                                       |
| <b>•• :•• •• (points 4-6, 4-5-6, 4-5, 2-5) inverted circumflex accent</b> |                                    |                                                                                       |
| ◁ :° :: °° (points 4-6, 5, 4-6, 2-5)                                      | <b>positive circular arrow</b>     | *                                                                                     |
| • • :•° (points 4-6, 5, 4-5-6, 2-5)                                       | <b>tilde</b>                       |                                                                                       |
| (points 3-6)                                                              | <b>minus</b>                       | -                                                                                     |

|                                                   |                                                                  |    |
|---------------------------------------------------|------------------------------------------------------------------|----|
| •: (points 2-5-6)                                 | point                                                            |    |
| •: •: •: (points 2-5-6, 2-5-6, 2-5-6)             | ellipsis                                                         |    |
| °i (points 2-6)                                   | subscript                                                        | Q, |
| . °. (points 6, 2-6)                              | subscript on the left <b>in chemistry</b>                        |    |
| °. °↑ (points 2-6, 2-6)                           | subscript                                                        | □  |
| • • • • •<br>• • (points 4-5, 2-6)                | <b>PPCM</b> ; inclusive <b>disjunction</b>                       | v  |
| . ° (points 3-5)                                  | multiplied by                                                    | x  |
| . ° . ° (points 3-5, 3-5)                         | multiplicative point; scalar product                             |    |
| • . ° (points 5, 3-5)                             | asterisk, star; convolution product                              | •  |
| • • • • •<br>• • • (points 5, 5, 3-5)             | large multiplication cross                                       | ×  |
| • • •<br>• • • (points 4-6, 3-5)                  | <b>tensor product</b>                                            | ⊗  |
| . • • • (points 4-6, 4-6, 3-5)                    | large tensor product                                             | ⊗  |
| • • • • •<br>• • • • • (points 4-5-6, 4-5-6, 3-5) | double cross product                                             | ⊗  |
| • • •<br>• • • (points 4-5-6, 3-5)                | factorial                                                        |    |
| • • •<br>• • • (points 4-5, 3-5)                  | <b>vector product; outer product;</b><br>GCD; conjunction; wedge |    |
| • • • • •<br>• • • • • (points 4-5, 4-5, 3-5)     | vector product                                                   | ∧  |

|                                                              |                                            |                  |
|--------------------------------------------------------------|--------------------------------------------|------------------|
| $\therefore^\circ$ (points 2-3-5)                            | greater                                    | $\overset{+}{+}$ |
| $\bullet \therefore$ (points 5, 2-3-5)                       | unity                                      |                  |
| $\therefore \bullet$ (points 4-5-6, 2-3-5)                   |                                            |                  |
| $\therefore \therefore \bullet$ (points 4-5-6, 4-5-6, 2-3-5) | great union                                | $\cup$           |
| $\dots$                                                      |                                            |                  |
| $\bullet \therefore$ (points 4-5, 2-3-5)                     | intersection                               |                  |
| $\bullet \bullet \therefore$ (points 4-5, 4-5, 2-3-5)        | large intersection                         | $\cap$           |
| $\therefore \bullet$ (points 4-6, 2-3-5)                     | plus in a circle                           | $\oplus$         |
| $\succ \therefore^\circ$ (points 4-6, 4-6, 2-3-5)            | large plus in a circle                     | $\bigoplus$      |
| $\therefore \dots$ (points 2-3-5, 3-6)                       | more or less                               | $\pm$            |
| $\dots i^\circ$ (points 3-6, 2-3-5)                          | less or more                               | $\mp$            |
| $\therefore$ (points 2-3-6)                                  | opening parenthesis on a line large        | $(\dots$         |
| $\bullet \therefore$ (points 5, 2-3-6)                       | opening parenthesis<br>on a line (nesting) | $(\dots$         |
| $\bullet \therefore$ (points 4-5, 2-3-6)                     | opening parenthesis on several lines       |                  |
| $\therefore \therefore$ (points 4-6, 2-3-6)                  | opening brace on one line                  | $\{\dots$        |
| $\hat{\therefore} \therefore$ (points 4-5-6, 2-3-6)          | opening brace on multiple lines            |                  |

|                                  |                                                         |         |
|----------------------------------|---------------------------------------------------------|---------|
| .. (points 2-5-6)                | closing parenthesis on a line large closing parenthesis | ...)    |
| • ∴ (points 5, 2-5-6)            | parenthesis on a line (nesting)                         | ...)    |
| • ∴ (points 4-5, 2-5-6)          | closing parenthesis on multiple lines                   |         |
| ∴ ∴ (points 4-6, 2-5-6)          | closing brace on one line closing brace                 | ...}    |
| ∴ ∴ (points 4-5-6, 2-5-6}        | on multiple lines                                       | ...}    |
| • ∴ (points 1-2-3-5-6)           | opening bracket on one line                             | [...]   |
| • ∴ (points 5, 1-2-3-5-6)        | large opening bracket on one line (nesting)             | [...]   |
| • ∴ (points 4-5, 1-2-3-5-6)      | opening bracket on multiple lines                       |         |
| ∴ ∴ (points 4-6, 1-2-3-5-6)      | double opening bracket on one line                      | [[...]] |
| ∴ ∴ (points 2-3-4-5-6)           | closing bracket on one line                             | ...]    |
| • ∴ (points 5, 2-3-4-5-6)        | large closing bracket on one line (nesting)             | ...]    |
| • ∴ (points 4-5, 2-3-4-5-6)      | hook closing on several lines double                    |         |
| ∴ ∴ (points 4-6, 2-3-4-5-6)      | hook closing on one line                                | ...]]   |
| ∴ ∴ (points 2-3-5-6)             | equal                                                   |         |
| ∴ ∴ (points 4-6, 2-3-5-6)        | different                                               |         |
| ∴ ∴ (points 5, 2-3-5-6)          | from                                                    |         |
|                                  | slightly different from                                 |         |
| ° • ∴ (points 4-5-6, 5, 2-3-5-6) | double underlined equivalence sign                      |         |
| • ∴ (points 4-5, 2-3-5-6)        | equivalent to; asymptotically equal to                  | -       |

OR = OR TO

|                                                                       |                                        |               |
|-----------------------------------------------------------------------|----------------------------------------|---------------|
| $\therefore$ ii (points 4-5-6, 2-3-5-6)                               | equivalence underlined twice           |               |
| $\therefore$ i i: (points 2-3-5-6, 2-3-5-6)                           | congruent with;<br>identical to        | - OR $\equiv$ |
| $\therefore$ ii i: (points 4-6, 2-3-5-6, 2-3-5-6)                     | its negation                           | e or z        |
| $\circ \circ \therefore$ (points 2-5, 2-3-5-6)                        | corresponds to                         |               |
| $\therefore$ (points 3-4-6)                                           | SİFIUS                                 | sin           |
| $\therefore$ .. (points 4-6, 3-4-6)                                   | <b>cosine</b>                          | COS           |
| • .. (points 4-5, 3-4-6)                                              | arc sine                               | arc sine      |
| $\therefore \circ \therefore$ .. (points 4-5, 4-6, 3-4-6)             | arc cosine                             | £IFCCOS       |
| • .. (points 5, 3-4-6)                                                | percent                                | %             |
| $\therefore \circ \therefore$ .. (points 5, 3-4-6, 3-4-6)<br>thousand | per                                    | ‰             |
| $\therefore$ (points 2-3-4-6)                                         | tangent                                | tan           |
| $\therefore$ i. (points 4-6, 2-3-4-6)                                 | cotangent                              | cotan         |
| • i. (points 4-5, 2-3-4-6)                                            | arc tangent                            | arctan        |
| $\therefore \circ \therefore$ i. (points 4-5, 4-6, 2-3-4-6)           | arc cotangent                          | arccotan      |
| $\therefore$ (points 1-2-3-4-6)                                       | integral, sign sum                     | $\int$        |
| $\therefore$ (points 4-6, 1-2-3-4-6)                                  | curvilinear integral                   | $\oint$       |
| $\therefore \circ \therefore$ (points 1-2-3-4-6, 1-2-3-4-6)           | double integral                        | $\iint$       |
| $\therefore$ t. (points 4-6, 1-2-3-4-6, 1-2-3-4-6)                    | double integral<br>on a closed surface | $\oiint$      |

|                                                  |                                          |                            |
|--------------------------------------------------|------------------------------------------|----------------------------|
| $\vdots \ddots$ (points 4-5-6, 1-2-3-4-6)        | sum after circular permutation           | $y$                        |
| $\langle \bullet \ddots$ (points 1-2-3-4-5-6)    | vertical bar on a line                   | $ \dots ou \dots $         |
| $\cdot \ddots$ (points 4-5-6, 1-2-3-4-5-6)       | vertical bar on several lines            | OR                         |
| $\bullet \ddots$ (points 4-5, 1-2-3-4-5-6)       | double vertical bar on a line            | $\parallel$ or $\parallel$ |
| $\bullet \cdot \ddots$ (points 4-6, 1-2-3-4-5-6) | double vertical bar<br>on multiple lines | OR                         |

#### IV. Symbols obtained by combinations of some keys and letters

|                                                |                    |            |
|------------------------------------------------|--------------------|------------|
| $\bullet$ (points 4-5, 1-4)                    | infinity           | $\infty$   |
| $\bullet \cdot$ (points 5, 1-4-5)              | partial derivative | $\partial$ |
| $\cdot \bullet \cdot$ (points 4-6, 4-5, 1-4-5) | laplacien          | $\Delta$   |

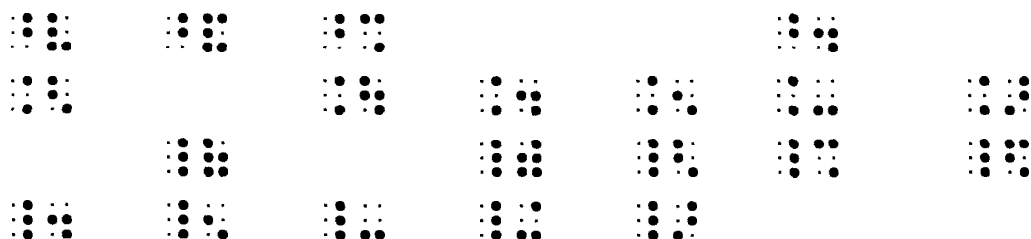


## V. Available symbols

This list is not exhaustive. It allows for the transcription of mathematical symbols that do not have a corresponding symbol in Braille in this notation. All symbols that have a meaning in the Braille Transcription Code for Printed Texts (1st edition, January 2006) are excluded.

### a) The "basic" list

This list includes the available combinations of two characters that are best suited for constructing additional mathematical symbols.



### b) Some symbols formed from two identical characters



### c) Symbols formed from letters preceded by the key: °

### d) Other symbols based on the punctuation marks "comma," "semicolon," "apostrophe"

